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ON THE STABILITY OF THE STATIONARY ROTATIONS OF A SYMMETRIC RIGID BODY IN AN ALTERNATING MAGNETIC FIELD*

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The stability of the stationary rotations of a conducting axisymmetric rigid body with a fixed centre of mass in a magnetic field is studied. The field is assumed to be homogeneous and of fixed direction, and to vary in strength harmonically. The principal axes of polarization are assumed to be the same as the principal axes of inertia of the body. The problem is analysed by using a small parameter, proportional to the square of the amplitude of the magnetic field and inversely proportional to the kinetic moment of the body, by the method of special matrix transformations /1, 2/. As distinct from the well-known method, see e.g., /3/, the present method retains its efficiency in the quite general case, notably, when the spectrum of the limit matrix has identically multiple points, so that any resonance situations can be studied. The conditions for stability of the stationary rotations of the body are obtained, and a domain of stability is found in parameter space when there is a resonance relation between the frequencies of the magnetic field and the nutational oscillations of the body.

1. Formulation of the problem. We consider a conducting symmetric rigid body, having a fixed point O, the same as the centre of mass of the body. Let $O\xi_1\xi_2\xi_3$, $Ox_1x_2x_3$ be right orthogonal trihedrons with origin at the point O. The ξ_i axes have a fixed orientation in space, the x_i axes are directed along the principal axes of inertia of the body, while the x_3 axis coincides with the axis of symmetry of the body.

We assume that the magnetic field is uniform and that the projections of its field-strength vector $\mathbf{H}_\xi = \mathbf{H}_\xi(t)$ onto the ξ_i axes are

$$H_{\xi_1} = H_{\xi_2} = 0, \quad H_{\xi_3} = H_0 \sin \omega t \tag{1.1}$$

Here, $H_0 = \text{const}$ is the modulus of the field-strength vector, and ω is the field frequency.

We assume that the depth of field penetration into the conducting material is much greater then the size of the body (i.e., the frequency ω in (1.1) is not very high), and we take the permeability μ of the body to be unity. Then the principal term of the asymptotic expansion of the moment of the forces acting on the body in the uniform field is /4/

$$\mathbf{M} = \mathbf{H}_\xi \Gamma A^\circ \Gamma^T ([\mathbf{H}_\xi, \mathbf{\Omega}_\xi] + \mathbf{H}_\xi^\cdot) \tag{1.2}$$

Here, $\mathbf{\Omega}_\xi$ is the instantaneous angular velocity vector, A° is the polarization tensor in the x_i axes, $\Gamma = \|\gamma_{ij}\|$, γ_{ij} is the cosine of the angle between the ξ_i and x_j axes, and T denotes transposition; the dot denotes differentation with respect to time.

We assume that the principal axes of the body inertia tensor are at the same time the principal axes of the polarization tensor, i.e., in (1.2) we have $A^\circ = \operatorname{diag}\{\alpha_1, \alpha_2, \alpha_3\}$, α_i are the polarization coefficients with respect to the x_i axes, while $\alpha_1 = \alpha_2$, $\alpha_i = \text{const}$.

Under these assumptions, the equations of motion of the body about the fixed point O in the field (1.1) under the action of the moment of forces (1.2) are

$$L_1^\cdot = -H^2\left[\frac{\alpha_1}{I_1} L_1 + \left(\frac{\alpha_1}{I_3} - \frac{\alpha_1}{I_1}\right) L_{x_3}\gamma_1\right] - \tag{1.3}$$
$$H^2\left(\frac{\alpha_3}{I_1} - \frac{\alpha_1}{I_1}\right) Q_3\gamma_2 - (\alpha_3 - \alpha_1) HH^\cdot \gamma_2\gamma_3 \equiv \Phi(L_1, L_{x_3}, Q_3, \gamma_1, \gamma_2)$$
$$L_2^\cdot = \Phi(L_2, L_{x_3}, Q_3, \gamma_2, \gamma_1), \quad L_3^\cdot = 0$$
$$\gamma_i^\cdot = Q_i/I_1 \quad (i = 1, 2, 3), \qquad Q_1 = L_2\gamma_3 - L_3\gamma_2 \qquad (1\ 2\ 3)$$

Here, L_i is the angular momentum about the ξ_i axis, $\gamma_i \equiv \gamma_{i_3}$ is the projection onto the

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ξ_i axis of the unit vector γ, directed along the axis of symmetry of the body, x_3, I_i are the moments of inertia about the axes x_i $(I_1 = I_2)$, $L_{x_3} = L_1\gamma_1 + L_2\gamma_2 + L_3\gamma_3$ is the angular momentum about the x_3 axis, and (1 2 3) denotes clockwise permutation of the subscripts.

System (1.3) has the particular solution

$$L_1 = L_2 = 0, \quad L_3 = L = \text{const}, \quad \gamma_1 = \gamma_2 = 0, \quad \gamma_3 = 1 \tag{1.4}$$

It corresponds to stationary rotation about the axis of symmetry x_3, the same as the direction along which the field-strength vector (1.1) varies. On linearizing system (1.3) in the neighbourhood of the singular point (1.4) and retaining the same notation for the deviations of the variables $L_1, L_2, \gamma_1, \gamma_2$ from their stationary values (1.4), we arrive at a system of linear differential equations with periodic coefficients

$$\begin{aligned} &L_1^{\cdot} = -\alpha_1 H^2 [L_1 - (1 - I_1/I_3)\, L\gamma_1]\, I_1^{-1} - (\alpha_3 - \alpha_1)\, HH^{\cdot}\gamma_2 \equiv \\ &\Psi(L_1, \gamma_1, \gamma_2), \quad L_2^{\cdot} = \Psi(L_2, \gamma_2, \gamma_1) \\ &\gamma_1^{\cdot} = (L_2 - L\gamma_2)/I_1, \quad \gamma_2^{\cdot} = (L\gamma_1 - L_1)/I_1 \end{aligned} \tag{1.5}$$

To analyse system (1.5) it is best to introduce the complex-valued functions $\gamma = \gamma_1 + i\gamma_2$, $l = (L_1 + iL_2)/I_1$ of the real variable t and to make the replacement $\gamma = x_1 + x_2$, $l = \Omega x_2$. As a result, we reduce system (1.5) to the form

$$\begin{aligned} &\mathbf{x}^{\cdot} = [A_0 + \varepsilon A_1(t)]\,\mathbf{x}, \quad \varepsilon = \alpha_1 H_0^2/L \\ &\mathbf{x} = \begin{Vmatrix} x_1 \\ x_2 \end{Vmatrix}, \quad A_0 = \Lambda_0 = \begin{Vmatrix} i\Omega & 0 \\ 0 & 0 \end{Vmatrix}, \quad A_1(t) = \begin{Vmatrix} -p(t) & -r(t) \\ p(t) & r(t) \end{Vmatrix} \\ &r(t) = -\chi\Omega \sin^2 \omega t + {}^1/_2 i\omega(\xi - 1)\sin 2\omega t \\ &p(t) = \Omega \sin^2 \omega t + r(t), \quad \chi = I_1/I_3, \quad \xi = \alpha_3/\alpha_1, \quad \Omega = L/I_1 \end{aligned} \tag{1.6}$$

Here, Ω is the frequency of the nutational oscillations of the body.

In actual systems the dimensionless parameter ε is small compared to unity, so that we shall consider the stability of the trivial solution of system (1.6) for sufficiently small ε.

2. The method of special matrix transformations /1, 2/. We consider in R^n the system of linear differential equations with a small parameter ε and T-periodic coefficients

$$\begin{aligned} &\mathbf{x}^{\cdot} = A(t, \varepsilon)\,\mathbf{x}, \quad \mathbf{x} = \text{col}\,(x_1, x_2, \ldots, x_n) \\ &A(t, \varepsilon) = \sum_{k=0}^{\infty} A_k(t)\,\varepsilon^k \quad (|\varepsilon| \leqslant \varepsilon_0), \quad A_0 = \text{const} \end{aligned} \tag{2.1}$$

of which a particular case is system (1.6).

For system (2.1) we have:

Theorem 1. Let the spectrum $\{\lambda_{0j}\}_1^n$ of the constant matrix A_0 satisfy the conditions

$$\begin{aligned} &\lambda_{0j} - \lambda_{0k} \neq i2\pi q/T \\ &(j \neq k; \quad j, k = 1, 2, \ldots, n; \quad q = 0, \pm 1, \pm 2, \ldots) \end{aligned} \tag{2.2}$$

Then, for sufficiently small ε, there is a non-degenerate T-periodic transformation (E is the identity matrix)

$$\mathbf{x} = S(t, \varepsilon)\,\mathbf{y} \equiv [E + \varepsilon S_1(t) + \ldots + \varepsilon^N S_N(t)]\,\mathbf{y} \tag{2.3}$$

which reduces system (2.1) to the form

$$\mathbf{y}^{\cdot} = B(t, \varepsilon)\,\mathbf{y}, \quad B(t, \varepsilon) = \sum_{k=0}^{\infty} B_k(t)\,\varepsilon^k \tag{2.4}$$

where, for any N, the matrices $B_j = \Lambda_j$ $(j \leqslant N)$ are constant and diagonal.

Proof. By (2.2), it can be assumed without loss of generality that the matrix A_0 in (2.1) is diagonal, $A_0 = \Lambda_0$. The replacement (2.3) reduces Eq.(2.1) to Eq.(2.4), in which $B(t, \varepsilon) = S^{-1}(AS - S^{\cdot})$. Consequently, the unknown matrix $S(t, \varepsilon)$ satisfies the equation

$$S^{\cdot} = AS - SB \tag{2.5}$$

Given any square matrix $A = \| a_{jk} \|$, we introduce the notation $A^{(d)} = \text{diag}\,\{a_{11}, \ldots, a_{nn}\}$, $A^{(nd)} = A - A^{(d)}$. After substituting the expansions in powers of ε of matrices A, B, S, into Eq.(2.5) and comparing coefficients of like powers of ε, we arrive at the sequence of linear differential matrix equations

$$S_j^{\cdot} = \Lambda_0 S_j - S_j \Lambda_0 + P_j(t) - \Lambda_j \quad (j = 1, 2, \ldots, N) \tag{2.6}$$

Here,

$$P_1(t) = A_1(t)$$
$$P_j(t) = A_j(t) + \sum_{k=1}^{j-1} (A_{j-k}S_k - S_k\Lambda_{j-k}) \quad (j = 2, 3, \ldots, N)$$

We put

$$\Lambda_j = \frac{1}{T}\int_0^T P_j^{(d)}(t)\,dt, \quad S_j^{(d)}(t) = \int_0^t [P_j^{(d)}(t) - \Lambda_j]\,dt \tag{2.7}$$

The diagonal terms of the matrix $S_j(t)$ are thus defined.
The equation for the off-diagonal terms

$$S_j^{(nd)\cdot} = \Lambda_0 S_j^{(nd)} - S_j^{(nd)}\Lambda_0 + P_j^{(nd)}(t) \tag{2.8}$$

spolits into $n^2 - n$ scalar equations

$$s_{kl}^{\cdot} = \beta_{kl}s_{kl} + p_{kl}(t), \quad \beta_{kl} \equiv \lambda_{0k} - \lambda_{0l} \quad (k \neq l)$$

each of which has the T-periodic solution

$$s_{kl}(t) = \frac{\exp[\beta_{kl}(t+T)]}{1 - \exp(\beta_{kl}T)} \int_t^{t+T} \exp(-\beta_{kl}\tau)\,p_{kl}(\tau)\,d\tau \tag{2.9}$$

By successively evaluating by means of (2.7) and (2.9) the diagonal matrices Λ_j and the elements $s_{kl}(t)$ of the T-periodic matrices $S_j(t)$, we can find transformation (2.3) and system (2.4), which it was required to prove.

Now assume that the spectrum of the matrix A_0 has multiple points and that

$$\lambda_{0j} - \lambda_{0k} \neq i2\pi q/T$$
$$(j \neq k;\ j, k = 1, 2, \ldots, p; \quad 1 \leqslant p < n;\ q = 0, \pm 1, \quad \pm 2, \ldots) \tag{2.10}$$

As above, we shall assume that A_0 has been reduced to the Jordan form, i.e.,

$$A_0 = J_0 = \operatorname{diag}\{J_{01}, \ldots, J_{0p}\}; \quad J_{0j} = \lambda_{0j}E + M_{0j}$$
$$(j = 1, 2, \ldots p),$$

where M_{0j} are known nilpotent matrices. Introducing fractional powers of $\varepsilon = \varepsilon_1^m$ ($m = \operatorname{LCM}_j m_j$; $m_j = \dim J_{0j}$) by means of the replacement

$$\mathbf{x} = N(\varepsilon_1)\mathbf{y}; \quad N(\varepsilon_1) = \operatorname{diag}\{1, \varepsilon_1^{m/m_1}, \ldots, \varepsilon_1^{(m_1-1)m/m_1}, \ldots, 1, \varepsilon_1^{m/m_p}, \ldots, \varepsilon_1^{(m_p-1)m/m_p}\} \tag{2.11}$$

we obtain the system (E are identity matrices of suitable dimensions)

$$\mathbf{y}^{\cdot} = B(t, \varepsilon)\mathbf{y}; \quad B_0 = \Lambda_0 = \operatorname{diag}\{\Lambda_{01}, \ldots, \Lambda_{0p}\} \quad \Lambda_{0j} = \lambda_{0j}E \quad (j = 1, 2, \ldots, p).$$

Noting the structure of the matrix J_0, we denote the block diagonal part of any square matrix $A = \|A_{jk}\|$ by $A^{(d)} = \operatorname{diag}\{A_{11}, \ldots, A_{pp}\}$ and accordingly $A^{(nd)} = A - A^{(d)}$, where $\dim A_{jj} = \dim J_{0j}$. By using the T-periodic non-degenerate replacement for sufficiently small $\varepsilon_1 > 0$

$$\mathbf{y} = S(t, \varepsilon_1)\mathbf{v}; \quad S(t, \varepsilon_1) = E + \sum_{k=1}^{N} S_k(t)\varepsilon_1^k \tag{2.12}$$

we can obtain the system

$$\mathbf{v}^{\cdot} = P(t, \varepsilon)\mathbf{v}; \quad P(t, \varepsilon) = \sum_{k=0}^{\infty} P_k(t)\varepsilon_1^k \tag{2.13}$$

where $P_j(t) = C_j^{(d)}$ $(j = 1, 2, \ldots, N)$ are constant block diagonal matrices. Each matrix $S_j(t)$ of (2.12) then satisfies the differential equation

$$S_j^{\cdot} = Q_j(t) - C_j^{(d)} + \Lambda_0 S_j^{(nd)} - S_j^{(nd)}\Lambda_0 \tag{2.14}$$
$$Q_1(t) = P_1(t), \quad Q_j(t) = P_j(t) + \sum_{k=1}^{j-1} (P_{j-k}(t)S_k(t) - S_k(t)C_{j-k}^{(d)}) \quad (j = 2, 3, \ldots, N)$$

By solving Eqs.(2.14) we can find successively all the matrices

$$C_j^{(d)} = \frac{1}{T}\int_0^T Q_j^{(d)}(t)\,dt, \quad S_j^{(d)}(t) = \int_0^T [Q_j^{(d)}(t) - C_j^{(d)}]\,dt,$$

where the matrices $S_j^{(nd)}$ are found from the equations

$$S_j^{(nd)\cdot} = \Lambda_0 S_j^{(nd)} - S_j^{(nd)}\Lambda_0 + Q_j^{(nd)}(t) \quad (j = 1, 2, \ldots, N).$$

As a result, for any N, system (2.13) splits, up to $O(\varepsilon_1^{N+1})$, into p subsystems of the type

$$\mathbf{v}_k^{\cdot} = [\lambda_{0k}E + \varepsilon_1 C_{1k} + \ldots + \varepsilon_1 C_{Nk} + O(\varepsilon_1^{N+1})]\,\mathbf{v}_k$$
$$(k = 1, 2, \ldots, p),$$

where C_{jk} are constant matrices.

If the matrix C_{1k} is reduced to diagonal form, the required result is obtained by means of the transformation

$$\mathbf{v}_k = \left(E + \sum_{k=1}^{N} F_{jk}\varepsilon_1^{k}\right)\mathbf{w}_k \quad (k = 1, 2, \ldots, p)$$

If C_{1k} is reduced to the Jordan form, on introducing new fractional powers of ε_1, we can make another step after transformation (2.11) and repeat the above arguments. We assume that, in each block, at some finite step (which may be distinct for distinct blocks), the corresponding matrix has a simple spectrum. In view of this, we can assert:

Theorem 2. Let the matrix A_0 in system (2.1) have multiple points in its spectrum, which satisfy conditions (2.10). Then, under our assumptions, for sufficiently small $\varepsilon > 0$, there is a non-degenerate T-periodic transformation which reduces system (2.1) to the form (2.4), where the parameter ε may be different.

In the resonant case, when, for certain fixed j, k, q, we may have

$$\lambda_{0j} - \lambda_{0k} = i2\pi q/T,$$

the matrix Λ_0 has to be written as $\Lambda_0 = N_0 + iR_0$, where the spectrum of the matrix N_0 satisfies condition (2.2), while

$$R_0 = \operatorname{diag}\{2\pi q_1/T, \ldots, 2\pi q_m/T\}$$
$$(q_j = 0, \pm 1, \pm 2, \ldots; j = 1, 2, \ldots, m)$$

The replacement $\mathbf{x} = \exp(iR_0 t)\,\mathbf{v}$ then reduces system(.21) to the form

$$\mathbf{v}^{\cdot} = Q(t, \varepsilon)\,\mathbf{v}; \quad Q(t, \varepsilon) = N_0 + \sum_{k=1}^{\infty} Q_k(t)\,\varepsilon^k \tag{2.15}$$

where condition (2.2) holds for system (2.15), so that Theorem 1 or 2 can be applied.

Note. On comparing our procedure with the well-known methods of analysing equations with periodic coefficients, we can observe that there is in /5/ one of the so-called indirect methods of studying stability, based on finding the characteristic exponents $\alpha(\varepsilon)$, found as "implicit functions of ε from the equation $\Phi(\alpha, \varepsilon) = 0$" (/5/, pp.249-291). Our method of special matrix transformations /1, 2/ is a "direct" method, whereby a system with an almost constant, and moreover, almost diagonal matrix, can be found after relatively simple transformations. In a sense, our procedure can be regarded as an asymptotic analogue of the Floquet-Lyapunov theorem on the reducibility of systems of ordinary differential equations.

3. Analysis of the stability conditions for stationary rotations of a rigid body. Let us apply the above method to Eq.(1.6). By (2.7), in the non-resonant case Eq. (1.6) transforms to

$$\mathbf{y}^{\cdot} = [\Lambda_0 + \varepsilon\Lambda_1 + O(\varepsilon^2)]\mathbf{y} \tag{3.1}$$
$$\Lambda_1 = \frac{1}{T}\int_0^T A_1^{(d)}(t)\,dt = \frac{\Omega}{2I_3}\begin{Vmatrix} I_1 - I_3 & 0 \\ 0 & -I_1 \end{Vmatrix}$$

Consequently, for a rigid body with an oblate ellipsoid of inertia $(I_3 > I_1)$, the trivial solution is asymptotically stable for sufficiently small ε and $\Omega \neq 2\omega$. If the ellipsoid of inertia is prolate $(I_3 < I_1)$, it follows from (3.1) that the stationary rotation is unstable.

Resonance occurs in the system when the frequency of nutational oscillations is close to twice the magnetic field frequency, i.e.,

$$\Omega = 2\omega + \varepsilon\Omega_1 \tag{3.2}$$

The constant parameter Ω_1 in (3.2), on which the solution depends, will be called the detuning. Substituting (3.2) into (1.8), we make the replacement

$$\mathbf{x} = \exp(\Lambda_0 t)\,\mathbf{v}.$$

The variable v satisfies the equation

$$\mathbf{v}^{\cdot} = \varepsilon Q_1(t)\,\mathbf{v};\quad Q_1(t) = \exp(-\Lambda_0 t)\,A_1 \exp(\Lambda_0 t) = \begin{Vmatrix} -p(t) + i\Omega_1 & -r(t)\exp(-2i\omega t) \\ p(t)\exp(i2\omega t) & r(t) \end{Vmatrix} \tag{3.3}$$

System (3.3) belongs to the class of systems (2.1) with one Jordan cell and a multiple zero root, so that, by Theorem 2, it can be transformed by means of the replacement (2.12) to the form

$$\mathbf{y}^{\cdot} = [\varepsilon P_1 + O(\varepsilon^2)]\,\mathbf{y} \tag{3.4}$$

$$P_1 = \begin{Vmatrix} (1-\chi)\omega + i\Omega_1 & \omega(1 - 2\chi - \xi)/4 \\ -\omega(1 - 2\chi + \xi)/4 & -\chi\omega \end{Vmatrix}$$

For system (3.4) the characteristic equation is quadratic with complex coefficients, dependent on χ, ξ, Ω_1. Its roots lie in the left half-plane when

$$\left(\frac{\chi - {}^1/_2}{\sqrt{(4+\zeta)/[4(3+\zeta)]}}\right)^2 + \left(\frac{\xi}{\sqrt{4+\zeta}}\right)^2 < 1 \tag{3.5}$$

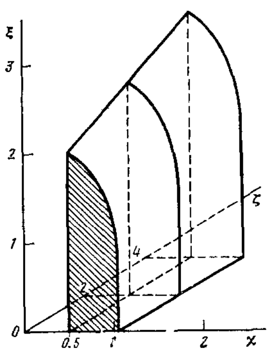

Fig.1

In space of parameters $\chi, \xi, \zeta = 4\Omega_1^2/\omega^2$ (for actual bodies ${}^1/_2 < \chi < \infty$, $0 < \xi$, $0 < \zeta$), condition (3.5) defines the domain of stability (Fig.1). The section of domain (3.5) by the plane $\zeta =$ const is an ellipse with centre on the line $\chi = {}^1/_2$, $\xi = 0$. For zero detuning, i.e., when $\zeta = 0$, the semi-axes of the ellipse (3.5) are equal respectively to $1/\sqrt{3}, 2$, and, if $I_1 < 1.077\, I_3$, it is possible to stablize the rotation of a body with a slightly prolate ellipsoid of inertia. The semi-axis in the plane $\xi = 0$ decreases monotonically as the detuning increases and tends asymptotically to the value ${}^1/_2$. In the plane $\chi = {}^1/_2$ the semi-axis increases without limit, so that, for sufficiently large values of the detuning Ω_1, there are no constraints on the ratio ξ of the polarization coefficients. Thus, for large detunings, the domain of stability (3.5) transforms into the domain obtained in the non-resonant case: $I_1 < I_3$ $(\chi < 1)$, the ellipsoid of inertia is compressed, and the polarization coefficients can take any values (ξ is arbitrary).

Notice that, in /6/, where the motion of a conducting rigid body with resonant interaction with an alternating magnetic field was considered, no asymptotically stable stationary mode of the body was discovered, and the hypothesis was put forward that it is very unlikely that the body will be captured in a resonant rotation. In the present paper, where, as distinct from /6/, the polarization tensor can be arbitrary, a domain is found where stable stationary modes are possible.

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BRANCHING AND STABILITY OF PERMANENT ROTATIONS AND RELATIVE EQUILIBRIA OF A BODY SUSPENDED FROM A ROD*

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The problem of the motion of a rigid body with a triaxial central ellipsoid of inertia suspended from a fixed point of a weightless non-deformable rod whose point of contact with the body lies on the principal central axis of inertia, is considered. Sets of all permanent rotations and relative equilibria of the body, their branching and stability, are studied. The results are presented in the form of bifurcation diagrams. The distribution of permanent rotations (relative equilibria) on these diagrams obeys the law of variation of stability when the value of the area integration constant (the angular velocity of the translational rotation of the body) is fixed.

The permanent rotations and relative equilibria of a body suspended on a string were studied in /1-3/.

1. Let us consider the motion of a body suspended on a hinge from a weightless non-deformable rod attached to a fixed point O_1 with the point of suspension O lying on the principal central axis of inertia.

The equations of motion of the body admit of energy and area integrals, and we have the following expression /4/ for the changed potential energy of the system:

$$W = {}^1/_2\, k^2 J^{-1} + \Pi$$
$$\Pi = -mg(l\mathbf{v} - \mathbf{a})\cdot\boldsymbol{\varkappa}, \quad J = \boldsymbol{\varkappa}\cdot\boldsymbol{\Theta}\cdot\boldsymbol{\varkappa} + m[\boldsymbol{\varkappa}\times(l\mathbf{v} - \mathbf{a})]^2$$

Here k is the constant of the area integral, Π is the potential energy due to gravity, J is the moment of inertia of the body about the vertical passing through the point O_1, m and $\boldsymbol{\Theta}$ is the mass and central tensor of inertia of the body with diagonal elements J_1, J_2, J_3, $\boldsymbol{\varkappa}$ and $\mathbf{v}$ are unit vectors of the descending vertical and the direction of the string from the point O_1 to O, $\mathbf{a}$ is the radius vector of the point O relative to the centre of mass C of the body, and g and l are the acceleration due to gravity and the length of the rod.

We introduce two right rectangular coordinate systems: the system $Cx_1x_2x_3$ rigidly attached to the body, whose axes coincide with the principal central axes of inertia, and the system $O_1y_1y_2y_3$ rotating with angular velocity $\Omega = kJ^{-1}$ about the y_3 axis directed vertically downwards.

We shall assume that the point O at which the rod is joined to the body, lies on the x_3 axis whose direction coincides with the direction of the vector $\mathbf{a}$. We shall denote by ν_s the projections of the vector $\mathbf{v}$ on to the y_s $(s = 1, 2, 3)$ axes. Let $\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\gamma}$ be the unit vectors of the x_1, x_2, x_3 axes and $\alpha_s, \beta_s, \gamma_s$ their projections on to the y_s axes, and

$$\pi_\alpha = \boldsymbol{\alpha}^2 - 1 = 0, \quad \pi_\beta = \boldsymbol{\beta}^2 - 1 = 0, \quad \pi_\gamma = \boldsymbol{\gamma}^2 - 1 = 0, \quad \pi_\nu = \mathbf{v}^2 - 1 = 0 \tag{1.1}$$
$$\pi_{\alpha\beta} = \boldsymbol{\alpha}\cdot\boldsymbol{\beta} = 0, \quad \pi_{\beta\gamma} = \boldsymbol{\beta}\cdot\boldsymbol{\gamma} = 0, \quad \pi_{\gamma\alpha} = \boldsymbol{\gamma}\cdot\boldsymbol{\alpha} = 0$$

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